＜完全楕円積分＞

第一種完全楕円積分　complete elliptic Integral of the first kind

定義

$$K(k)=\int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{1-k^2\sin^2\theta}}d\theta \qquad (1)$$

k:複素数　ただしk≠1　　数百桁精度

$$K(k)=\int_0^1\frac{1}{\sqrt{(1-t^2)(1-k^2t^2)}}dt$$

$K(0.3791)=$1.63230831743087802744821063444457519914622411009602905722187402832788274783300355825695525843365 6025

$K(\sqrt{2.5+i})=1.15514506065693 + 0.952845371467053i$

$K(0)=1.5707963267949$

$K(1)=\infty$

第二種完全楕円積分　complete elliptic Integral of the second kind

定義

$$E(k)=\int_0^{\pi/2}\sqrt{1-k^2\sin^2\theta}\,d\theta$$

k:　複素数　　数百桁精度

$$E(k)=\int_0^1\frac{\sqrt{1-k^2t^2}}{\sqrt{1-t^2}}dt$$

$E(0)=1.5707963267949$

$E(0.5)=$1.46746220933942715545979526699091613602536175232723196050079063649082422727129063565403853073350 4602

$E(1)=1$

$E(\sqrt{3+2.5i})=1.19974884333427 - 1.35709743760095i$

第三種完全楕円積分　complete elliptic Integral of the third kind

定義

$$\Pi(n,k)=\int_0^{\pi/2}\frac{1}{(1-n\sin^2\theta)\sqrt{1-k^2\sin^2\theta}}d\theta$$

n,k:複素数　　ただし　n,k≠1

数百桁精度

$$\Pi(n,k)=\int_0^1 \frac{1}{1-nt^2}\frac{1}{\sqrt{(1-t^2)(1-k^2t^2)}}dt$$

$\Pi(0.8,0.9)=5.98207408136457$

$\Pi(0.8,1)=\infty$

$\Pi(1,0.9)=\infty$

$\Pi(0.4,0.6+2i)=1.21173770101995 + 0.191364800453331i$

$\Pi(0.8+0.5i,0.9)=2.49425441175117 + 2.18227756380894i$

$\Pi(0.8+0.5i,0.9)=2.4942544117511747105522896591064314520301894913213 + 2.1822775638089445151863868582002698918277881413858i$

$\Pi(0.4,\sqrt{0.6})=2.59092115655522$

$\Pi(-2,1.27201964951407 - 0.786151377757423i)$
$=0.784049226661343 - 0.258414516193648i$

$\Pi(0.4,1)=\infty$