<ラプラス変換例>

プロフェッショナル版限定機能

いずれも代数計算で実行します

$$\mathcal{L}\{1\}=\frac{1}{s}$$

$$\mathcal{L}\{a\}=\frac{a}{s}$$

$$\mathcal{L}\left\{\frac{1}{2}\right\}=\frac{1}{2s}$$

$$\mathcal{L}\{t^2\}=\frac{2}{s^3}$$

$$\mathcal{L}\{t^5\}+\mathcal{L}\{t^2\}=\frac{2s^3+120}{s^6}$$

$$\mathcal{L}\{t^2+t^5\}=\frac{120}{s^6}+\frac{2}{s^3}$$

$$\mathcal{L}\{-t^5\}=-\frac{120}{s^6}$$

$$\mathcal{L}\{t^2-t^5\}=-\frac{120}{s^6}+\frac{2}{s^3}$$

$$\mathcal{L}\{(t^2+t^5)\}=\frac{120}{s^6}+\frac{2}{s^3}$$

$$\mathcal{L}\{e^{-t}\}=\frac{1}{s+1}$$

$$\mathcal{L}\{t^2\}-\mathcal{L}\{e^{-t}\}=\frac{-s^3+2s+2}{s^4+s^3}$$

$$\mathcal{L}\{2e^{-at}\}=\frac{2}{a+s}$$

$$\mathcal{L}\{\sin 2.34567894t\}=\frac{2.34568}{s^2+5.50221}$$

$$\mathcal{L}\{\cos at\}=\frac{s}{a^2+s^2}$$

$$\mathcal{L}\{\sin at\}=\frac{a}{a^2+s^2}$$

$$\mathcal{L}\{\sin 2t-\cos 4t\}=-\frac{s}{s^2+16}+\frac{2}{s^2+4}$$

$$\mathcal{L}\{te^{-at}\}=\frac{1}{(a+s)^2}$$

$$\mathcal{L}\{t^3e^{-at}\}=\frac{6}{(a+s)^4}$$

$$\mathcal{L}\{t^2e^{-at}\}=\frac{2}{(a+s)^3}$$

$$\mathcal{L}\{t^{n-1}e^{-at}\}=\frac{(n-1)!}{(a+s)^n}$$

$$\mathcal{L}\{t^{k-1}e^{-at}\}=\frac{\Gamma(k)}{(a+s)^k}$$

$$\mathcal{L}\{\sqrt{t}\}=\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$$

$$\mathcal{L}\left\{\frac{1}{\sqrt{t}}\right\}=\frac{\sqrt{\pi}}{s^{\frac{1}{2}}}$$

$$\mathcal{L}\left\{\frac{1}{\sqrt{t^3}}\right\}=\mathcal{L}\left\{\frac{1}{\sqrt{t^3}}\right\}$$ 解はありません。求まってはいけないケースです。

$$\mathcal{L}\left\{\frac{e^{-at}}{2}\right\}=\frac{1}{2(a+s)}$$

$$\mathcal{L}\left\{\frac{e^{-at}}{b-a}\right\}=\frac{-1}{(a+s)(a-b)}$$

$$\mathcal{L}\left\{\frac{e^{-at}}{b-a}\right\}-\mathcal{L}\left\{\frac{e^{-bt}}{b-a}\right\}=\frac{1}{ab+as+bs+s^2}=\frac{1}{(b+s)(a+s)}$$

$$\mathcal{L}\left\{\frac{e^{-at}-e^{-bt}}{b-a}\right\}=\frac{-1}{(a+s)(a-b)}+\frac{1}{(b+s)(a-b)}=\frac{1}{(a+s)(b+s)}$$

$$\mathcal{L}\{t^5 e^{-3t}\}+\mathcal{L}\{\sin t\}$$
$$=\frac{s^6+18s^5+135s^4+540s^3+1335s^2+1458s+849}{s^8+18s^7+136s^6+558s^5+1350s^4+1998s^3+1944s^2+1458s+729}$$
$$=\frac{s^6+18s^5+135s^4+540s^3+1335s^2+1458s+849}{(s+3)^6(s^2+1)}$$

$$\mathcal{L}\left\{\frac{1}{a^2}(1-\cos at)\right\}=\frac{-s}{a^2(a^2+s^2)}+\frac{1}{a^2 s}=\frac{1}{s(a^2+s^2)}$$

$$\mathcal{L}\left\{\frac{1}{a^3}(at-\sin at)\right\}=\frac{1}{a^2 s^2}+\frac{-1}{a^2(a^2+s^2)}=\frac{1}{s^2(a^2+s^2)}$$

$$\mathcal{L}\left\{\frac{1}{2a^3}(\sin at-at\cos at)\right\}=\frac{a^2-s^2}{2a^2(a^2+s^2)^2}+\frac{1}{2a^2(a^2+s^2)}=\frac{1}{(a^2+s^2)^2}$$

$$\mathcal{L}\left\{\frac{\cos at-\cos bt}{b^2-a^2}\right\}=\frac{-s}{(a+b)(a-b)(a^2+s^2)}+\frac{s}{(a+b)(a-b)(b^2+s^2)}$$
$$=\frac{s}{(a^2+s^2)(b^2+s^2)}$$

$$\mathcal{L}\{t\cos at\}=\frac{-a^2+s^2}{(a^2+s^2)^2}$$

$$\mathcal{L}\{t\sin at\}=\frac{2as}{(a^2+s^2)^2}$$

仮想関数を使ったラプラス変換

$f(t)=\varnothing$ 　　　　 $g(t)=\varnothing$

$$\frac{d}{dt}\mathcal{L}\{f(t)\}=s\mathcal{L}\{f(t)\}-f(0)$$

$$\frac{d}{dt}\mathcal{L}\{g(t)\}=s\mathcal{L}\{g(t)\}-g(0)$$

$$\frac{d^2}{dt^2}\mathcal{L}\{f(t)\}=s^2\mathcal{L}\{f(t)\}-(sf(0)+f'(0))$$

$$\mathcal{L}\{\sinh at\}=\frac{-a}{(a+s)(a-s)}$$

$$\mathcal{L}\{\sinh at-(t^2-t^5)\}=\frac{120}{s^6}-\frac{2}{s^3}+\frac{-a}{(a+s)(a-s)}$$

$$\mathcal{L}\{\cosh at\}=\frac{-s}{(a+s)(a-s)}$$

$$\mathcal{L}\{1-\cos at\}=-\frac{s}{a^2+s^2}+\frac{1}{s}$$

$\mathcal{L}\{t^{-2}e^{-at}\}=\mathcal{L}\{t^{-2}e^{-at}\}$ 解はありません。求まってはいけないケースです。

$\mathcal{L}\{t^{-1}e^{-at}\}=\mathcal{L}\{t^{-1}e^{-at}\}$ 解はありません。求まってはいけないケースです。

$$\mathcal{L}\{L_n(t)\}=\frac{(s-1)^n}{ss^n}=\frac{(s-1)^n}{s^{n+1}}$$

$$\mathcal{L}\{L_3(t)\}=\frac{s^3-3s^2+3s-1}{s^4}$$

$$\mathcal{L}\left\{\frac{1}{t+a}\right\}=e^{as}E_1(as) \qquad a>0$$

$$\mathcal{L}\left\{\frac{1}{(t+a)^2}\right\}=\frac{e^{as}E_2(as)}{a} \qquad a>0$$

$$\mathcal{L}\left\{\frac{1}{(t+a)^n}\right\}=a^{1-n}e^{as}E_n(as) \qquad a>0 \qquad n=0,1,2,3...$$

$$\mathcal{L}\left\{\frac{1}{t^2+1}\right\}=\frac{1}{2}\pi\cos s+\mathrm{Ci}(s)\sin s-\mathrm{Si}(s)\cos s$$

$$\mathcal{L}\{J_0(at)\}=\frac{1}{\sqrt{a^2+s^2}}$$

$$\mathcal{L}\{J_1(at)\}=\frac{-s+\sqrt{a^2+s^2}}{a\sqrt{a^2+s^2}}$$

$$\mathcal{L}\{J_k(at)\}=\frac{\left(-s+\sqrt{a^2+s^2}\right)^k}{a^k\sqrt{a^2+s^2}} \qquad k>0$$

$$\mathcal{L}\left\{\frac{1}{t}J_k(at)\right\}=\frac{\left(\sqrt{s^2+a^2}-s\right)^k}{a^k k} \qquad k>0$$

$$\mathcal{L}\left\{t^{k-\frac{1}{2}}J_{k-\frac{1}{2}}(at)\right\}=\frac{2^{-\frac{1}{2}+k}a^{-\frac{1}{2}+k}\Gamma(k)}{ks^2\sqrt{\pi}+a^2k\sqrt{\pi}} \qquad k>0$$

$$\mathcal{L}\{\sin at\sinh at\}=\frac{2a^2s}{(s^2-2as+2a^2)(s^2+2as+2a^2)}$$

$$\mathcal{L}\{\sin at-at\cos at\}=\frac{a}{s^2+a^2}-\frac{as^2-a^3}{(s^2+a^2)^2}$$

$$\mathcal{L}\{t^2\sin at\}=\frac{6a^2s^2-2a^4}{(s^2+a^2)^3}$$

$$\mathcal{L}\left\{\frac{\sin kt}{t}\right\}=\tan^{-1}\frac{k}{s}$$

$$\mathcal{L}\left\{\frac{1}{t}(1-\cos at)\right\}=\frac{1}{2}\ln\frac{s^2+a^2}{s^2}$$

$$\mathcal{L}\left\{\frac{1}{t}(1-\cosh at)\right\}=\frac{1}{2}\ln\frac{s^2-a^2}{s^2}$$

$$\mathcal{L}\left\{e^{-\frac{t^2}{4k^2}}\right\}=e^{k^2s^2}k\sqrt{\pi}\,\mathrm{erfc}(ks)$$

$$\mathcal{L}\left\{\operatorname{erf}\left(\frac{t}{2k}\right)\right\}=\frac{e^{k^2s^2}\operatorname{erfc}(ks)}{s}$$

$$\mathcal{L}\left\{\frac{1}{\sqrt{t}(t+k)}\right\}=\frac{e^{ks}\pi\operatorname{erfc}(\sqrt{ks})}{\sqrt{k}}$$

$$\mathcal{L}\{\operatorname{Si}(kt)\}=\frac{\tan^{-1}\frac{k}{s}}{s}$$

$$\mathcal{L}\left\{\frac{1}{\sqrt{t^3}}e^{-at}\right\}=-2\sqrt{s\pi+a\pi}$$

$$\mathcal{L}\left\{\frac{1}{\sqrt{t^2-k^2}}H(t-k)\right\}=K_0(ks)$$

$$\mathcal{L}\left\{\frac{1}{t}e^{-\frac{k^2}{4t}}\right\}=2K_0(k\sqrt{s})$$

$$\mathcal{L}\{\sqrt{t(t+2k)}\}=\frac{e^{ks}kK_1(ks)}{s}$$

$$\mathcal{L}\{H(t-k)\}=\frac{e^{-ks}}{s}$$

$$\mathcal{L}\{H(t)\}=\frac{1}{s}$$

$$\mathcal{L}\{(t-k)^{n-1}H(t-k)\}=\frac{e^{-ks}\Gamma(n)}{s^n} \qquad n>0$$